32/30 [redacted] [redacted]

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STT 200 Section 20[redacted] 12:40 Midterm Exam

Unless requested, do not evaluate answers.

1. A random variable X has the following probability distribution:

x	0	1	5
p(x)	0.4	0.6	0.2

a. E X

$0 + 0.6 + 1 = 1.6$ ✓

b. E X^2

$0 + 0.6 + 5^2 \cdot 0.2 = 5.6$ ✓ ✓

c. Var X

$5.6 - (1.6)^2 = 5.6 - 2.56 = 3.04$

d. Standard deviation of X

$\sqrt{3.04} = 1.7436$ ✓

2. A random variable X has E X = 8, variance X = 4. Denote by T the random **total** of 900 independent plays of X.

a. E T

$8 \cdot 900 = 7200$ ✓

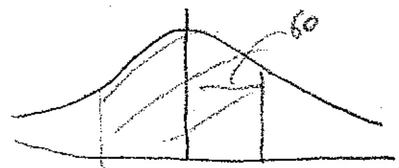
b. Var T

$4 \cdot 900 = 3600$ ✓

c. Standard deviation of T

$\sqrt{3600} = 60$ ✓

d. Sketch the approximate distribution of total T, clearly indicating the means and standard deviation of T as recognizable elements in your sketch.



e. Determine the standard score z of total = 7360.

$Z\text{score} = \frac{7360-7200}{60} = \frac{160}{60} = 2.6667$

given A ∴ 0.4962
Table

3. Events A, B have

P(A) = 0.2 $A^c = 0.8$

P(B | IF A) = 0.6

P(B | IF A^C) = 0.3

a. P(A and B)

$0.2 \cdot 0.6 = 0.12$

b. P(A^C and B)

$0.8 \cdot 0.3 = 0.24$

c. P(B)

$0.2 \cdot 0.6 + 0.8 \cdot 0.3 = 0.12 + 0.24 = 0.36$

d. P(A | IF B)

$\frac{P(A \text{ and } B)}{P(B)} = \frac{0.12}{0.36} = 0.3333$

4. In terms of constants a, b, c and Var X, Var Y of independent random variables X, Y,

a. E (a X − b Y + c)

$a E(X) - b E(Y) + c$ ✓

b. Var(a X − b Y + c)

$a^2 \text{Var} X + b^2 \text{Var} Y$ ✓

5. Box 1 has 4R 4G
Box 2 has 3R 9G
Box 1 will be chosen with probability 0.7
Box 2 will be chosen with probability 0.3

A ball will be selected with equal probability from the box chosen.

a. Intuitively, is P(Box 1 $|_{IF}$ R) or P(Box 1) the larger? Why?

ϕa

$P(\text{Box1}) = 0.7$ so $P(\text{Box1} |_{IF} R) > P(\text{Box1})$

$P(\text{Box1} |_{IF} R) = 0.8235$ is larger.

b. P(Box 1 and R)

$0.7 \cdot \frac{4}{8} = \frac{7}{10} \frac{1}{2} = 0.35$ ✓

c. P(R)

$0.7 \cdot \frac{4}{8} + 0.3 \cdot \frac{3}{12} = 0.35 + 0.075 = 0.425$ ✓

d. P(Box 1 $|_{IF}$ R).

$\frac{P(\text{Box1 and R})}{P(R)} = \frac{0.35}{0.425} = 0.8235$ ✓

6. Calculator may be used.

x	(x-Mean[x])^2	x^2
2	$\frac{324}{25}$	4
4	$\frac{64}{25}$	16
4	$\frac{64}{25}$	16
8	$\frac{144}{25}$	64
10	$\frac{484}{25}$	100
—	—	—
28	$\frac{216}{5}$	200

(totals at bottom)

a. Standard deviation σ of list x.

$\frac{43.20}{5} = \sigma^2 = 8.64 \quad \sigma = 2.9394$

b. Median of list x.

4

c. Standard deviation σ of list -5x + 8.

$5 \cdot \sigma = \sigma = 14.697$

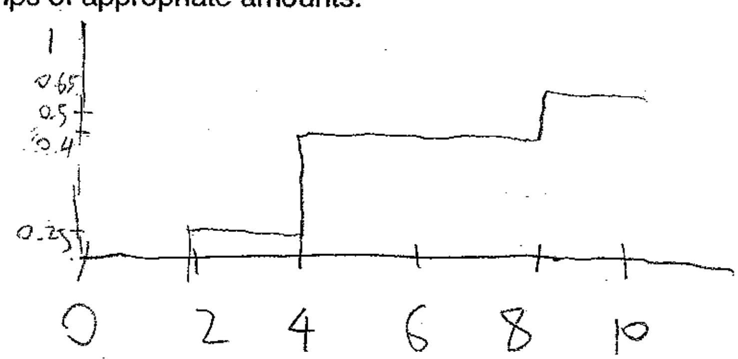
d. Median of list -5x + 8.

-12

e. Height of the probability histogram for list x over the interval [3, 9].

$\frac{\frac{3}{5}}{9-3} = \frac{\frac{3}{5}}{6} = 0.1$

f. Sketch the cumulative probability distribution for list x. Be sure it rises between 0 and 1 with jumps of appropriate amounts.

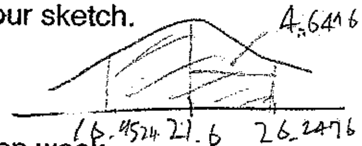


$\mu = 21.6$

$\sigma = \sqrt{21.6} = 4.6476$

7. Poisson. We expect on average 21.6 falcon sightings per week and the number of sightings is thought to follow a Poisson distribution. Recall that for the Poisson the mean and variance are the same.

a. Sketch the approximate distribution of the Poisson for this case. Label the numerical mean and standard deviation as recognizable numerical elements of your sketch.



b. Give a 68%% interval for the number of sightings in a given week.

9.2952

c. The formula for Poisson p(x) is $e^{-\mu} \frac{\mu^x}{x!}$ for x = 0, 1, ...
p(20)

$= e^{-21.6} \cdot \frac{(21.6)^{20}}{20!}$

(first write all appropriate numbers in the formula, evaluate the factorial, then evaluate using calculator).

8. Binomial. Each time a casting from production is x-rayed there is probability 0.1 it will be found defective. These events are thought to be pretty much independent.

a. The probability that the first six castings x-rayed are:
def not-def def def def non-def

$0.1 \cdot 0.9 \cdot 0.1 \cdot 0.1 \cdot 0.1 \quad 0.9 = (0.1)^4 \cdot (0.9)^2 = 0.000081$

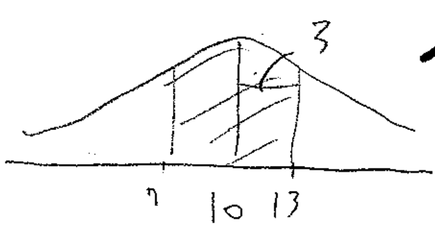
b. The number of ways to select four of six positions as the only ones defective.

$_6C_4 = \frac{6!}{2! \cdot 4!} = 15$

c. The binomial probability that out of six castings there are precisely four that are found defective when x-rayed.

$15 \cdot (0.1)^4 \cdot (0.9)^2 = 0.0012$

d. Sketch the approximate distribution of the number X of castings, out of 100 castings x-rayed, that are found defective. Identify the mean and standard deviation of X numerically in your sketch.



$\mu = 100 \cdot 0.1 = 10$

$\sigma^2 = 100 \cdot 0.1 \cdot 0.9 = 9$

$\sigma = 3$